"HOLLOW" FUNCTIONS OF A SOURCE IN PROBLEMS OF RADIATION GASDYNAMICS

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§1. It is well-known that many models and methods used for the solution of spectroscopy problems of a steady and quasisteady optically dense plasma can be transferred with insignificant changes also to radiation gasdynamics [1-3]. At the same time, during the analysis of typical gasdynamic situations (cylindrical and spherical shock waves, flow around a body by a radiating gas or plasma, dynamic skin-effect [4-6], etc.), requirements often arise for the inclusion of specific models. This, in particular, refers to an extensive class of source functions, characterized by the presence of a "cavity" in the central region of a volume of gas or a plasma that is not completely transparent and that is radiating in a given range of frequencies. The "cavity" can also be a body, screening the central part of the luminous space being considered. Of course, among the traditional problems of plasma spectroscopy may be mentioned those which require the introduction into the calculation of "hollow" source functions. A typical example is that of discharges in inert gases with the displacement to the periphery of the luminous zone of atoms [7].

As far as it is known, this class of source functions up to now has not been investigated and the special features of the formation of spectral line contours corresponding to it (and also the integral characteristics found by means of them) have not been explained. The purpose of this paper is an attempt to fill this gap to a certain degree.

§2. Let us consider a circular section of a plasma volume (Fig. 1) in which three concentric zones can be distinguished with a certain radial line of vision for determinacy intersecting all three zones. We stipulate that the maximum of the source function is located within the bounds of zone 2 and that zone 1 is partially transparent. The points l_1 and l_3 correspond to the boundary of zone 3: Beyond its limits, absorption and emission of light in the frequency range being investigated cannot be taken into account. The point l_2 corresponds to the center of the chord $l_1l_2 = l_2l_3$ and l is some arbitrary point lying in any of the three zones.

We introduce the dimensionless coordinate x,

$$(ll_3/l_1l_2) = 2x + 1,$$

the optical thicknesses

$$\tau_x = \int_0^x \varkappa(\nu, x')\,dx',\ \ \tau = 2\int_0^{1/2} \varkappa(\nu, x')\,dx'$$

($\varkappa$ is the local absorption coefficient), and the dimensionless optical scale $\xi = \tau_x/\tau$. At the point l_1, $x = \xi = 1/2$; at l_2, $x = \xi = 0$; at l_3, $x = \xi = -1/2$; and at all intermediate points l, $x \neq \xi$.

We shall use the conventional light approximation [8], for which the transfer equation relative to the intensity (energy brightness) $I(\nu, x)$ has the form

$$(dI(\nu,\ x)/d\tau) = \Phi(\nu,\ x) - I(\nu,\ x), \tag{2.1}$$

where $\Phi = (\varepsilon(\nu, x)/\varkappa(\nu, x))$ is the source function (ε is the emission coefficient).

It can be verified [9] that the formal solution of Eq. (2.1) has the form

$$Y(\tau) = 2\tau\exp(-\tau/2)\int_0^{1/2} \varphi(\nu, x)\,\mathrm{ch}(\tau, \xi)\,d\xi, \tag{2.2}$$

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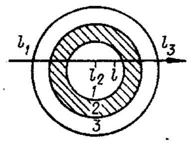

Fig. 1

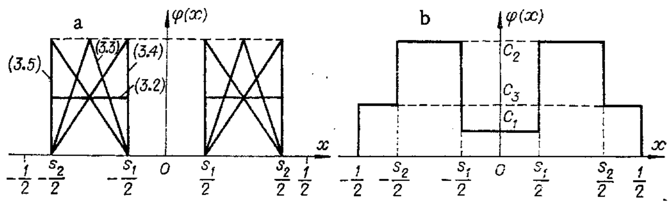

Fig. 2

where $\varphi(\nu, x) = \Phi(\nu, x)/B_l(\nu_0, T)$; $Y(\tau) = I(\nu)/B(\nu_0, T)$. Here $B(\nu_0, T)$ is the Planck function with frequency ν_0 corresponding to the center of the line and to some effective temperature T.

In this problem, actual knowledge of T and, consequently, also of $B(\nu_0, T)$ is not required: The Planck radiator in the calculations plays only the role of a certain threshold calibration standard [1].

§3. The general properties of the solution (2.2) for the condition that $x \neq \xi$ when the frequency ν is conserved in the form of the argument of the dimensionless source function $\varphi(\nu, x)$ are studied in fairly great detail in [9]. It is shown by what method the generalization of the results obtained with a considerably more simple approach should be derived when Eq. (2.2) is degenerated into a simple integral expression:

$$Y(\tau) = 2\tau \exp(-\tau/2) \int_0^{1/2} \varphi(x) \operatorname{ch}(\tau x)\, dx. \tag{3.1}$$

In our case it is also natural to start from formula (3.1). Moreover, based on comparisons of the different types of models [9], it is found to be possible to work with simple parametric functions of step, transcendental, and triangular types; in this case $Y(\tau)$ is represented in the form of several simple analytic expressions.

A. Rectangular Model (Fig. 2a). We determine $\varphi(x)$ in the form

$$\varphi(x) = \begin{cases} 0 & 0 \leqslant |x| \leqslant s_1/2, \\ C & s_1/2 \leqslant |x| \leqslant s_2/2, \\ 0 & s_2/2 \leqslant |x| \leqslant 1/2 \end{cases}$$

and we find $C = (s_2 - s_1)^{-1}$ from the condition of normalization

$$\int_0^{1/2} \varphi(x)\, dx = 1/2.$$

Formula (3.1) gives

$$Y(\tau) = 2(s_2 - s_1)^{-1} \exp(-\tau/2) [\operatorname{sh}(\tau s_2/2) - \operatorname{sh}(\tau s_1/2)]. \tag{3.2}$$

The choice of the rectangular model corresponds to the supposition that the second zone is homogeneous; i.e., it luminesces like a Planck radiator, and zones 1 and 3 contain only atom-absorbents. In this case $s_2 \geq s_1$, the boundaries of zone 2 are moved, and s_1 and s_2 can be time-dependent. When $s_1 \to 0$ and assuming that $s_2 < 1$, we arrive at the well-known Newton–Bleeker model, which has been studied in detail in [10].

B. Triangular Models (Fig. 2a). Conserving the same boundary parameters s_1 and s_2, taking into account the standardization of the function $\varphi(x)$, and fixing the apex of the triangle at the points $|x_0| = (s_1 + s_2)/4$ (isosceles triangle), $|x_0| = s_1/2$ and $|x_0| = s_2/2$ (right-angled triangles), we find, respectively,

$$Y(\tau, |x_0| = (s_1 + s_2)/4) = (16/\tau)(\exp(-\tau/2)/(s_2 - s_1)^2 [\operatorname{ch}(\tau s_1/2) + \operatorname{ch}(\tau s_2/2) - 2\operatorname{ch}(\tau(s_1 + s_2)/4)]; \tag{3.3}$$

$$Y(\tau, |x_0| = s_1/2) = (8/\tau)(\exp(-\tau/2)/(s_2 - s_1)^2) [\operatorname{ch}(\tau s_2/2) - \operatorname{ch}(\tau s_1/2) - (\tau/2)(s_2 - s_1)\operatorname{sh}(\tau s_1/2)]; \tag{3.4}$$

$$Y(\tau, |x_0| = s_2/2) = (8/\tau)(\exp(-\tau/2)/(s_2 - s_1)^2) [\operatorname{ch}(\tau s_1/2) - \operatorname{ch}(\tau s_2/2) + (\tau/2)(s_2 - s_1)\operatorname{sh}(\tau s_2/2)]. \tag{3.5}$$

When the triangle degenerates into a Dirac δ-function ($s_1 = s_2 = s$) we have

$$Y(\tau) = \tau \exp(-\tau/2) \operatorname{ch}(\tau s/2).$$

In the case $s = 0$, when the δ-function of the second zone is displaced on the axis of the radiator, we obtain the well-known result in [9] corresponding to an infinitely thin glowing filament surrounded by a layer of absorbing atoms.

C. Stepped Three-Zonal Model (Fig. 2b). We generalize the rectangular model, taking into account the circumstance that zones 1 and 3 can be not only absorbing, but also capable of making a finite contribution to the energy flux. We shall assume, for simplicity, that each of the three zones are homogeneous:

$$\varphi(x)=\begin{cases}C_1 & 0\leqslant |x|<s_1/2,\\ C_2 & s_1/2\leqslant |x|\leqslant s_2/2,\\ C_3 & s_2/2\leqslant |x|\leqslant 1/2.\end{cases}$$

Of the three constants C_1, C_2, and C_3, in view of the condition of normalization, only two are independent. We introduce the ratios $\gamma_1=(C_1/C_2)\leq 1$ and $\gamma_2=(C_3/C_2)\leq 1$. We obtain

$$Y(\tau)=2\exp(-\tau/2)/(\gamma_2+s_2(1-\gamma_2)-s_1(1-\gamma_1))[\gamma_2\,\mathrm{sh}(\tau/2)+(1-\gamma_2)\,\mathrm{sh}(\tau s_2/2)-(1-\gamma_1)\,\mathrm{sh}(\tau s_1/2)]. \quad (3.6)$$

It s_1 tends to zero, we arrive at the class of stepped two-zonal models described in [9].

Transition from $Y(\tau)$ directly to the contour of the reabsorbed line $I(\nu)$ is effected by means of the relation

$$\tau=\tau_{\max}Q(\nu),$$

where $Q(\nu)$ is the normalized profile of the line radiated by an optically thin layer with a maximum at $\nu=\nu_0$; $\tau_{\max}$ is the optical thickness, calculated for $\nu=\nu_0$.

§4. Some results of the calculations of the function $Y(\tau)$ with different "hollow" source functions are shown on a logarithmic scale in Figs. 3 to 5.

The effect of the model type of the source function $\varphi(x)$ on the curve of the relative energy brightness $Y(\tau)$ with the boundary parameters $s_1=0.4$, $s_2=0.8$ is shown in Fig. 3, where the curves correspond to the number of the numerical formulas $Y(\tau)$: 1) (3.2); 2) (3.3); 3) (3.4); 4) (3.5); 5) (3.6); $\gamma_1=0.2$, $\gamma_2=0$; 6) (3.6); $\gamma_1=0$, $\gamma_2=0.2$; 7) (3.6); $\gamma_1=\gamma_2=0.2$.

It can be seen that in all cases, when $\gamma_2=0$, the overall nature of the function $Y(\tau)$ is preserved. In the presence of a peripheral radiating zone ($\gamma_2\neq 0$), with increase of optical thickness the curve $Y(\tau)$ changes sharply: Depression of the contour [8] is stopped and the energy brightness in the central part of the line reaches an asymptote:

$$Y_0=\gamma_2[\gamma_2+s_2(1-\gamma_2)-s_1(1-\gamma_1)]^{-1}$$

The "hyper-Planck" excess of intensity over a certain interval $\Delta\tau$, predicted in [9, 11], is noted on curves 4 and 6.

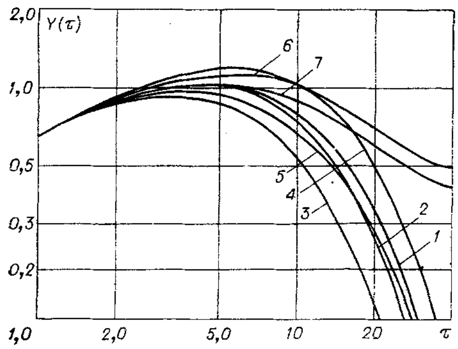

Fig. 3

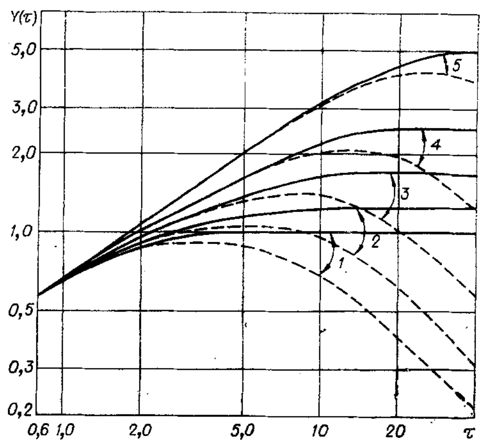

Fig. 4

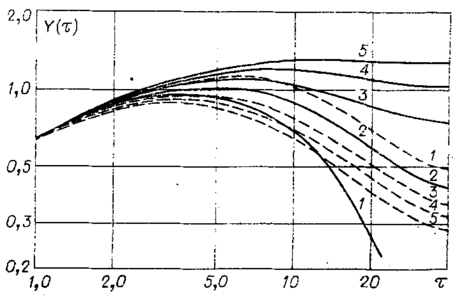

Fig. 5

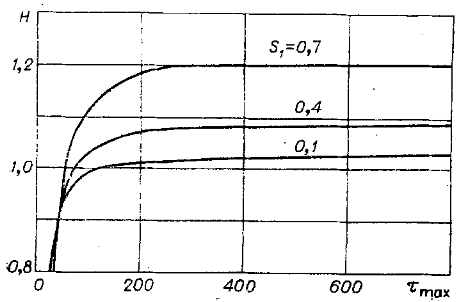

Fig. 6

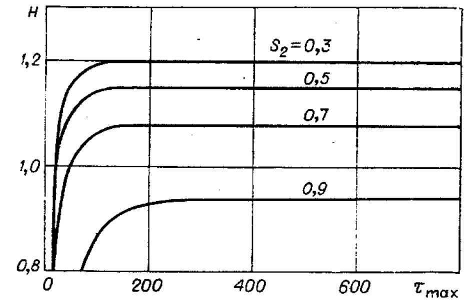

Fig. 7

In Fig. 4, two families of curves are compared which reflect the role of the parameters s_1 in shaping the $Y(\tau)$ curve for cases of a rectangular [solid line, formula (3.2)] and a triangular [dashed line, formula (3.3)] model of the source function, in which the parameter $s_2 = 1$, i.e., as if the self-pinched zone 3 does not exist. This same pattern is close to the case of a strong shock wave or a luminous skin-layer, forcing out a periodic absorbing shell. The parameter s_1 has been varied (0; 0.2; 0.4; 0.6; 0.8). It can be seen that with increase of τ for the two models of $\varphi(x)$, the curve $Y(\tau)$ is changed considerably. In one case emergence at a "Planck" or "hyper-Planck" asymptote is observed (rectangular), and in the other case the curves pass through a maximum and tend to zero, which corresponds to self-reversal of the line with an intensity at the apices which can exceed the Planck threshold (triangular). Applied to this same situation, the well-known pyrometric method of Bartels [12] must be modified.

Figure 5 shows the results of calculations of $Y(\tau)$ for the case of the three-zonal stepped model [formula (3.6)]. Curves with varying parameter γ_2 are shown by a solid line (0; 0.2; 0.4; 0.6; 0.8) and $\gamma_1 = 0.2$; curves with varying parameter γ_1 are shown by a dashed line (0; 0.2; 0.4; 0.6; 0.8) and $\gamma_2 = 0.2$. It can be seen that although the peripheral zone is twice as narrow as the central zone ($s_1 = 0.4$, $s_2 = 0.8$), its role in shaping of the reabsorbed line contours is considerably more important, and this can be understood from the general curves relating to the divergence of the radiant flux of a heterogeneous radiator [9, 11].

It is proposed to consider separately the problems of the direct shaping of the line contours, calculations of the integral spectral characteristics of heterogeneous optically dense objects with "hollow" source functions, and also the posing of the corresponding reverse (diagnostic) problems.

The role of the so-called "hollow" source functions in problems of radiation gasdynamics has been discussed above. A distinctive feature of these functions is the displacement of their maximum relative to the flux axis. The corresponding direct problems have been considered: The dependence was considered of the dimensionless energy brightness on the optical thickness of the layer, certain special features have been explained of the shaping of the contour of the emergent (reabsorbed) line, the causes of the occurrence of "hyper-Planck" surpluses of the energy brightness, etc. Below, one of the methods is proposed for estimating those parameters by which the dimensions and shape of the "cavity" are determined with the source function.

We shall confine ourselves to estimates of the boundary parameters s_1 and s_2 in the range of maximum optical thicknesses of the layer which are the most important from the applied point of view [13, 3]: $10^2 \leq \tau_{max} \leq 10^3$. In many problems of radiation gasdynamics it is important to determine not so much s_1 and s_2 individually, but rather their difference $\Delta s = s_2 - s_1$, since this quantity in explicit form occurs in both the radiation transfer equation and also in the energy equation (the term with the divergent radiation flux) [14]. We also note that because of the quite large values of τ_{max} stated above, the initial line profile can be assumed with good accuracy to be dispersed.

The success of the solution to the problem posed is determined in the main by the choice of the appropriate functional (quasiinvariant) H, which should be of low sensitivity to the quantity τ_{max} within a given range of $\Delta\tau_{max}$ and to the shape of the "hollow" source function profile, each of the halves of which is characterized by the width Δs. In addition, it is important that H should be fixed experimentally without special difficulties and with acceptable accuracy. The change of H with a variation of s_1 when the parameter s_2 is fixed and a variation of s_2 when the parameter s_1 is fixed should be as identical as possible.

The functional H of this type corresponds completely to the stated requirements: The region of the self-reversal trough of the emergent line contour, referred to the magnitude of the frequency range which divides the self-reversal maxima, is integrated. We note that a similar functional was considered in [15] in connection with another problem.

Figure 6 shows the function $H(\tau_{max})$, calculated on a computer, for the case when $s_2 = 0.8$ and s_1 increases from 0.1 to 0.7; Fig. 7 shows the same function, but for the conditions that $s_1 = 0.2$ and s_2 increases from 0.3 to 0.9.

It can be seen immediately from the results of the calculation given that the functional $H(\Delta s)$ can be recommended for the purpose of the diagnostics of plasma and hot gas streams in the presence of "hollow" source functions. In this case, small values of the difference Δs can be found more accurately; however, even for the range of values $0.5 \leq \Delta s \leq 0.9$ the systematic error of the determination does not exceed 5-10%.

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